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How quantum computation gates can be realized in terms of scattering theory approach to quantum tunneling of charge transport

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A B S T R A C T

Here it is shown that the quantum computation gates can be described in terms of electron transport through quantum tunneling. The two seemingly diverse areas where quantum gates are composed of rotation of electron spin and electronic charge transport is tunneling of electrons through impurity barriers are shown to be closely connected. The phase relation between the transmission and reflection amplitudes are important components which relate the tunneling phenomenon with the rotation of an effective spin in the Bloch sphere can be used for information processing through quantum gates.

Introduction

Two major areas of research that have evolved somewhat independently over the past three decades are phase coherent charge transport in mesoscopic [2–4] structures which are ultra-small structures smaller than the inelastic mean free path of an electron and quantum computing[1, 5, 6] based on coherent rotations of an electron spin. We have reviewed a theory of quantum scattering and notion of unitary scattering matrix to formulate quantum input-output relations. This differs from the standard approach to this subject in which the quantum gates for quantum computing are considered to be unitary time evolution operators. Here we have developed the effective spin concept to connects charge transport and spin qubit operation [7–10] to show how quantum computation gates[11–13] can be realized in terms of coherent charge transport [14, 15].

The motivation of the work here is to show that the representation of quantum gates as scattering matrices, S-matrices may have a physical realization. Here we study the one-dimensional scattering problem to describe tunneling charge transport. Tunneling problem is characterized by the scattering matrix [16] which is represented by

$$|\psi_{out}\rangle = \begin{pmatrix} b^+ \\ a^- \end{pmatrix} = \begin{pmatrix} t & r' \\ r & t' \end{pmatrix} \begin{pmatrix} a^+ \\ b^- \end{pmatrix} \equiv S|\psi_{in}\rangle \quad (1)$$

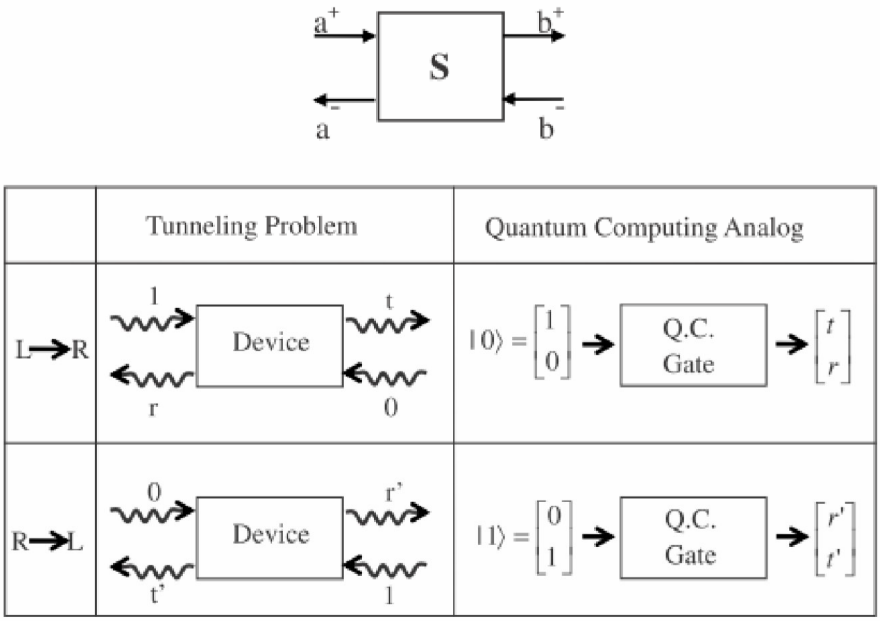

Fig.1 The tunneling problem and its quantum computing gate equivalent .The scattering matrix associated with the device relates the incoming (a^+ ,b^-) to the outgoing (a^-, b^+) wave amplitudes. It can be interrupted as the matrix representing the rotation of a qubit from the initial state $|\psi(IN)\rangle$to the final state$|\psi(OUT)\rangle$.

Classical computation

Classical computer is based on a collection of bits and gates which acts on an input state of bits to get the output state of bits. Bits of information, say, "0", with no current and "1" with current is logically manipulated in a circuit called a gate. Circuits of classical gates are well known, for example AND, OR and NOT gates are the basic gates which with combination produces the desired operation of more complex nature. Some classical gates are tabulated below. For example NOT gates, $NOT : A \rightarrow \overline{A}$; it means if $A = 0$ the $\overline{A} = 1$ and when $A = 1, \overline{A} = 0$.

Truth tables of gates, *OR* : $A \cup B$ and *AND* : $A \cap B$ are given together as

A	B	OR	AND
0	0	0	0
1	0	1	0
0	1	1	0
1	1	1	1

Quantum logic gates

For quantum case most general states are not $|0\rangle$ and $|1\rangle$ but general quantum bit or qubit is

$|\psi\rangle = a|0\rangle + b|1\rangle$. So a qubit has infinitely many states "in between" $|0\rangle$and $|1\rangle$. For two qubits we have states

$|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$. (2)

For $a = d = 0$ and $b = c = 1/\sqrt{2}$ we have $|\psi\rangle_{EPR} = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$ .This state possess a correlation between the two bits in such a way that it is not possible to generate by classical way. This phenomenon of entanglement between the bits bring the possibility of infinite information processing.

Practical difficulty in present day classical computing is as follows. Chip size is decreasing very fast in every decade from the first advent of computer since 1970 and to continue this rate it will reach down to the atomic dimension within 2020 and much before that electron-electron interference will be too prominent to use the circuits and enhancement of memory or CPU is not possible any further by just decreasing the size of the chip which necessitates the quantum computation (Moore's law). Universal quantum gates are used to produce any arbitrary operation as a composition of finite sequence of basic quantum gates from a desired set.

Equivalence between tunneling problem and quantum computing gate

An electron transport tunneling problem is completely characterized by the transmission and reflection amplitudes, (t,r) and (r',t'). For single qubit a quantum gate is a rotation operator of a spin vector on the Bloch sphere. Scattering matrix is unitary, i.e, $S^{\dagger}S = SS^{\dagger} = 1$ such that norm is conserved.

For the scattering from $L \rightarrow R$ an arbitrary output vector in the Bloch sphere can be written as

$|\Psi^{I}_{out}\rangle = e^{i\gamma}\left[\cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}\, e^{i\phi}|1\rangle\right]$ (3)

Where $|0\rangle \equiv \binom{1}{0}$ is north pole and $|1\rangle \equiv \binom{0}{1}$south pole of Bloch vector such that $|0\rangle$ and $|1\rangle$ are
mutually orthogonal, i.e, $\langle 0|1\rangle = 0$. The angles $(\theta,\square)$ are the azimuthal and polar angles, respectively and γ is an arbitrary phase factor. A coherent charge transport or tunneling in a mesoscopic device is represented as the successive set of rotations in the Bloch sphere.

Scattering across a single delta scatterer

We need to solve the Schrodinger equation, from figure 1one can get

$t' = t = \frac{i\tilde{k}}{i\tilde{k}-1}$ (4)

and

$r' = r = \frac{1}{i\tilde{k}-1}$ (5)

where– $\tilde{k} = k/k0$ with $k = \sqrt{2mE/\hbar^2}$ and $k_0 = \frac{mV}{\hbar^2}$. Here we have, $|t| = \frac{\tilde{k}}{\sqrt{\tilde{k}^2+1}}$ and

$\Phi T = -tan^{-1}(1/\tilde{k})$ and $|r| = \frac{1}{\sqrt{\tilde{k}^2+1}}$ and $\Phi_R = -tan^{-1}\tilde{k} - \pi$ and $\theta = \Phi_T - \Phi_R = -\pi/2$ and $\theta = 2tan^{-1}(1/\tilde{k})$.

For low energy $\theta = \pi$ as energy increases θ monotonically decreases to go to zero, a spin flip occurs, with $S_x = 0, S_y = -\hbar/2\frac{2\tilde{k}}{\tilde{k}^2+1}$ and $S_z = \frac{\hbar}{2}\left(\frac{\tilde{k}^2-1}{\tilde{k}^2+1}\right)$. When $\tilde{k} = 1$, Ψ_{out} is in equatorial plane and $\theta = \pi/2$ and

$U_{QG} = e^{-i\pi/4}.R_z(-\pi/2)R_y(\pi/2)R_z(+\pi/2) = e^{-i\pi/4}S(-\pi/2)\sigma_x HS(\pi/2)$ (6)

Where $S(\delta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix}$ is phase shift matrix and $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ is Hadamard matrix.

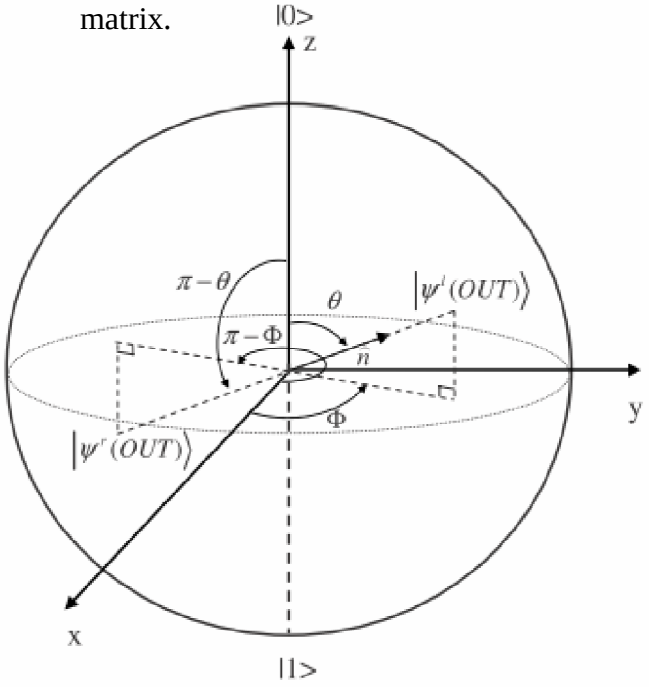

Fig 2. Bloch sphere representation of the effective spin (qubit) (OUT) The radius of the sphere is equal to 1.

$\Psi^l(OUT)\rangle(\Psi^r(OUT)\rangle)$ is the spinor given

in eq. (3) when the electron is incident from the left (right) contact.

Using similar examples like scattering through periodic and random arrays of N number of delta scatters produces localization of electrons. In this case resistance or conductance depends on the absolute value of the transmission amplitude. However, in this effective spin picture the transition from conductor to a insulator state corresponds to spin flipping which gives more information corresponding to other intermediate states involving the phase relation between the transmission and reflectance amplitudes.

Conclusion

Here we have shown how the quantum gates or quantum computation can be realized in terms of the tunneling devices. The correlation between the phase of the transmission and reflectance gives infinitely many conducting states corresponding to quantum conductance which can be used for the information processing.

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